HW 1.8: 8, 30, 36

**8.** Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

**-We can conclude there is a positive integer that equals the sum of the positive integers not exceeding it by providing the example of 1. Because the only positive integer that does not exceed 1 is itself, it proves the existence and the statement as true. This is a constructive proof.**

**30.** Prove that there are no solutions in integers *x* and *y* to the equation 2*x*2 + 5*y*2 = 14.

-1 ≤ y ≤ 1 (-1, 0, 1)

-2 ≤ x ≤ 2 (-2, -1, 0, 1, 2)

for (y, x)

(-1 , -2) = 13 **F**, (-1, -1) = 7 **F**, (-1, 0) = 5 **F**, (-1, 1) = 7 **F**, (-1, 2) = 13 **F**

(0, -2) = 8 **F**, (0, -1) = 2 **F**, (0, 0) = 0 **F**, (0, 1) = 2 **F**, (0, 2) = 8 **F**

(1, -2) = 13 **F**, (1, -1) = 7 **F**, (1, 0) = 5 **F**, (1, 1) = 7 **F**, (1, 2) = 13 **F**

**By exhausting all possibilities, we can conclude that there are no solutions in integers x and y to the equation 2*x*2 + 5*y*2 = 14.**

**36.** Prove that between every rational number and every irrational number there is an irrational number.

**-A real # r is rational if exists integers p and q with q ≠ 0 such that r = p/q.**

**- A real # r that is not rational is called irrational.**

**-x is an irrational number, and y is a rational number**

**-To find a value between two numbers, take their average. (x+y)/2**

**- y = s/t where t is not equal 0**

**To contradict:**

**- x = p/q where q is not equal to 0**

**-x+y/2 is equivalent to ((s/ø) + (p/ ø))/2**

**- Because our denominators (q and t) are not equal to zero, our averaged number will also be a rational number, due to the definition of a rational number.**

**- We thus have a contradiction of the initial proposition that x is an irrational number, proving that (x + y)/2 would result in an irrational number.**